

# A qualitative perspective on the dynamics of a single-Cooper-pair box with a phase-damped cavity

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In a recent paper Dajka, et.al., [J. Phys. A **40**, F879 (2007)] predicted that some composite systems can be entangled forever even if coupled with a thermal bath. We analyze the transient entanglement of a single-Cooper-pair box biased by a classical voltage and irradiated by a quantized field and find the unusual feature that the phase-damped cavity can lead to a long-lived entanglement. The results show an asymptotic value of the idempotency defect (concurrence) which embodies coherence loss (entanglement survival), independent of the interaction development by dependent critically on environment.

## 1 Introduction

Josephson junctions are being investigated as a possible route to scalable quantum computers [1]-[11]. The present lack of a current standard based on quantum devices has inspired several attempts to manipulate single electrons, where the rate of particle transfer is controlled by an external frequency. One of the physical realizations of a solid-state qubit is provided by a Cooper pair box which is a small superconducting island connected to a large superconducting electrode, a reservoir, through a Josephson junction [12]. Also realizations of superconducting charge qubits are a promising technology for the realization of quantum computation on a large scale [13]-[17].

In this context, a solid-state system is highly desirable because of its compactness, scalability and compatibility with existing semiconductor technology. Even though a Cooper pair box can contains millions of electrons at any one time, the box exhibits only two quantum charge states, depending upon whether or not a Cooper pair of electrons has recently tunneled into the box and various superconducting nanocircuits have been proposed as quantum bits (qubits) for a quantum computer [9, 10]. By gating the Cooper pairs into the box with an appropriate pulse width, previous research has shown that a coherent superposition of the two states can enable quantum computations. In architectures based on Josephson junctions coupled to resonators, the resonators store single qubit states, transfer states from one Josephson junction to another, entangle two or more

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Josephson junctions, and mediate two-qubit quantum logic. In effect, the resonators are the quantum computational analog of the classical memory and bus elements.

The present work is motivated by conjectures and statements presented in a recent fast track paper [18] and experimental results on Josephson junction and normal metal flux qubits coupled to the environment [6]. We obtain a long-lived entanglement using a superconducting charge qubit. More precisely, we endeavor to show the important property of entanglement via idempotency defect of a single Cooper pair box, due to the presence of a phase-damped cavity. Despite the complexity of the problem, we obtain a quite simple exact solution of the master equation that is valid for arbitrary values of the phase damping. In the framework of the exact solution of the master equation, we determine the coherence loss and the degree of entanglement. We perform a systematic analysis in order to reach an understanding of the Cooper pair dynamics in the presence of the decoherence. Physically, the effect of phase damping may be understood to be analogy of the  $T_2$  spin depolarization effects observed in nuclear magnetic resonance spectroscopy (for detailed physical motivation see Ref. [7]). Besides phase-damping-model importance in the description of different physical situations, it is very instructive since it allows for obtaining analytical treatments for different entanglement measures of some classes of states [8]. Some theoretical discussions and analysis of special cases of the problem at hand were given in Refs. [5, 19, 20] and experimental results were predicted in Ref. [6].

The organization of this paper is as follows: in section 2 we introduce the model and give the exact solution of the master equation. In section 3 we employ the analytical results obtained in section 2 to discuss the idempotency defect and entanglement for different values of the phase-damped cavity. Finally, we summarize the results in section 4.

## 2 The model

Several schemes have been proposed for implementing quantum computer hardware in solid state quantum electronics [21]. These schemes use electric charge, magnetic flux, superconducting phase, electron spin, or nuclear spin as the information bearing degree of freedom.

We start our analysis by presenting a brief discussion and a few physical principles of the Cooper pair box system. We consider a superconducting box with a low-capacitance Josephson junction with the capacitance  $C_J$  and Josephson energy  $E_J$ , biased by a classical voltage source  $V_g$  through a gate capacitance  $C_g$  and placed inside a single-mode microwave cavity. In particular, the schematic picture of this single-qubit structure may

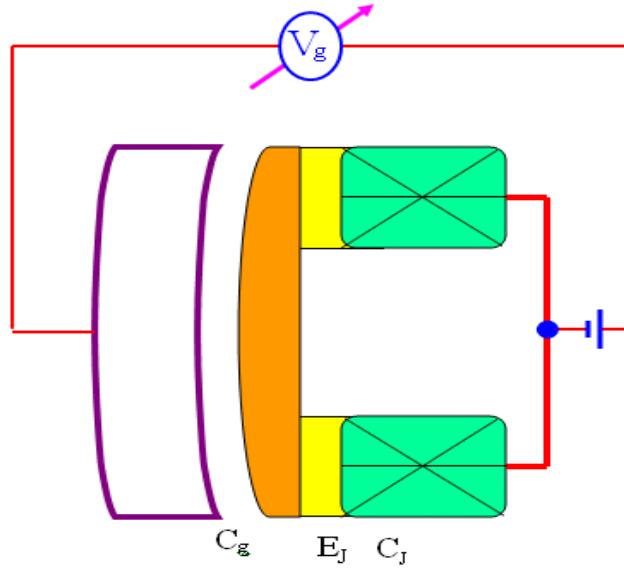


Figure 1: Schematic picture of the Cooper-pair box which is driven by an applied voltage  $V_g$  through the gate capacitance  $C_g$ . Black bars denote Cooper pair box. The two Josephson junctions have capacitance  $C_J$  and Josephson energy  $E_J$ . The driving microwave field is generated using the electrical voltage acting on the charge qubit via the gate capacitance [11].

be modeled as shown in figure 1. The total Hamiltonian of the system can be written as [22]

$$\hat{H} = \frac{1}{2}(Q - C_g V_g - C_g V)^2 (C_g + C_J)^{-1} - E_J \cos \phi + \hbar \omega (\psi^\dagger \psi + \frac{1}{2}),$$

where  $Q = 2Ne$  is the charge on the island ( $e$  is the electron charge and  $N$  is the number of Cooper-pairs) and  $\phi$  is the phase difference across the junction. The radiation field is to produce an alternating electric field of the same frequency across the junction, and  $V$  is the effective voltage difference produced by the microwave across the junction. We assume that the dimension of the device is much smaller than the wavelength of the applied quantized microwave (which is a realistic assumption), so the spatial variation in the electric field is negligible. We also assume that the field is linearly polarized, and is taken perpendicular to the plane of electrodes, then  $V$  can be written as [19, 23]  $V = i\hbar\omega (\hat{\psi} - \hat{\psi}^\dagger) / (2C_F)$ , where  $\hat{\psi}^\dagger$  and  $\hat{\psi}$  are the creation and annihilation operators of the microwave field with frequency  $\omega$ . We denote the capacitance parameter by  $C_F$ , which depends on the thickness of the junction, the relative dielectric constant of the thin insulating barrier, and the dimension of the cavity.

We consider the case where the charging energy with scale  $E_c = \frac{1}{2}e^2 (C_g + C_J)$ , which dominates over the Josephson coupling energy  $E_J$  and concentrates on the value  $V_g = e/C_g$ , so that only the low-energy charge states  $N = 0$  and  $N = 1$  are relevant. In this case the Hamiltonian, in the basis of the charge states  $|0\rangle$  and  $|1\rangle$ , reduces to a two-state form. In a spin-1/2 language [24]

$$\hat{H} = E_c (1 + e^{-2} C_J^2 V^2) - \frac{1}{2} E_J \sigma_x + 2e^{-1} E_c C_J V \sigma_z + \hbar \omega \left( \psi^\dagger \psi + \frac{1}{2} \right), \quad (1)$$

where  $\sigma_x$  and  $\sigma_z$  are the Pauli matrices in the pseudo-spin basis. It is to be noted that the charge states are not the eigenstates of the Hamiltonian (1), so the Hamiltonian can be diagonalized yielding the following two charge states subspace  $|e\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle)$  and  $|g\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$ . Here we employ these eigenstates to represent the qubit. If we consider a weak quantized radiation field and neglect the term containing  $V^2$ , the Hamiltonian (1) can be rewritten in the rotating wave approximations as

$$\hat{H} = \hbar \omega \psi^\dagger \psi + \frac{1}{2} E_J \sigma_z + \left\{ \frac{-i\hbar e C_J}{2(C_g + C_J)} \sqrt{\frac{\omega}{2\hbar C_F}} \psi \sigma_+ + H.c. \right\}. \quad (2)$$

We consider the interaction with an environment to be as the phase-damping type. This is a reservoir coupled to the field via the number operator of the indicating field, so that there is no energy damping, although there is a phase damping.

In order to obtain the general solution of the master equation for the density matrix under the phase damping of the cavity field at a zero temperature bath, we write

$$\frac{d\hat{\rho}(t)}{dt} = \frac{-i}{\hbar} [\hat{H}, \hat{\rho}(t)] + \gamma \left( 2\hat{\psi}^\dagger \hat{\psi} \hat{\rho}(t) \hat{\psi}^\dagger \hat{\psi} - \hat{\psi}^\dagger \hat{\psi} \hat{\psi}^\dagger \hat{\psi} \hat{\rho}(t) - \hat{\rho}(t) \hat{\psi}^\dagger \hat{\psi} \hat{\psi}^\dagger \hat{\psi} \right), \quad (3)$$

where  $\gamma$  is the phase-damping constant. An important feature of this quantized system is that its steady states, known as dressed states, are entangled. Switching to an interaction-picture representation for convenience by defining  $\hat{\rho}'(t) = \exp(i\hat{H}t)\hat{\rho}(t)\exp(-i\hat{H}t)$ , exact solution of equation (4) can be obtained in the dressed-states representations [25]. Consequently, equation (3) can be written as

$$\frac{d\hat{\rho}'(t)}{dt} = \gamma e^{i\hat{H}t} \left( 2\hat{\psi}^\dagger \hat{\psi} \hat{\rho}(t) \hat{\psi}^\dagger \hat{\psi} - \hat{\psi}^\dagger \hat{\psi} \hat{\psi}^\dagger \hat{\psi} \hat{\rho}(t) - \hat{\rho}(t) \hat{\psi}^\dagger \hat{\psi} \hat{\psi}^\dagger \hat{\psi} \right) e^{-i\hat{H}t}. \quad (4)$$

Next, we write the field operators  $\hat{\psi}$  and  $\hat{\psi}^\dagger \hat{\psi}$  in terms of the dressed states basis and get the initial state of the system expressed in the product density matrix forms. In the basis  $|n, e\rangle$  and  $|n, g\rangle$  states, the field operator  $\hat{\psi}$  can be written as  $\hat{\psi} = \sum_{n=0}^{\infty} \sqrt{n}(|n-1, e\rangle\langle n, e| + |n-1, g\rangle\langle n, g|)$ . Neglecting the oscillating terms of the master equation (4) in secular approximation, the density matrix in terms of the dressed states becomes

$$\begin{aligned} \frac{d\hat{\rho}'(t)}{dt} = & -\frac{\gamma}{4} \sum_{n=0}^{\infty} \left\{ \left( |\phi_n^{(+)}\rangle\langle\phi_n^{(+)}| \hat{\rho}'(t) |\phi_m^{(+)}\rangle\langle\phi_m^{(+)}| \right. \right. \\ & + |\phi_n^{(-)}\rangle\langle\phi_n^{(-)}| \hat{\rho}'(t) |\phi_m^{(-)}\rangle\langle\phi_m^{(-)}| + |\phi_n^{(+)}\rangle\langle\phi_n^{(+)}| \hat{\rho}'(t) |\phi_m^{(-)}\rangle\langle\phi_m^{(-)}| \\ & \left. \left. + |\phi_n^{(-)}\rangle\langle\phi_n^{(-)}| \hat{\rho}'(t) |\phi_m^{(+)}\rangle\langle\phi_m^{(+)}| \right) ((2n+1)^2 + 1) + |\phi_n^{(+)}\rangle\langle\phi_n^{(-)}| \hat{\rho}'(t) \right. \\ & \times |\phi_m^{(-)}\rangle\langle\phi_m^{(+)}| e^{2it\mu_{nm}} + |\phi_n^{(-)}\rangle\langle\phi_n^{(+)}| \hat{\rho}'(t) |\phi_m^{(+)}\rangle\langle\phi_m^{(-)}| e^{-2it\mu_{nm}} \\ & \left. + \frac{\gamma}{2} \sum_{n,m=0}^{\infty} \left\{ ((2n+1)(2m+1)(\phi_{nm}^{+-}) \hat{\rho}'(t) + \hat{\rho}'(t) \phi_{nm}^{+-}) \right\} \right\}, \quad (5) \end{aligned}$$

where  $|\phi_n^{(\pm)}\rangle$  are two eigenstates of the Hamiltonian (3) for a lossless cavity,  $\phi_{nm}^{(+ -)} = |\phi_n^{(+)}\rangle\langle\phi_n^{(+)}| + |\phi_m^{(+)}\rangle\langle\phi_m^{(+)}|$  and  $\mu_{nm} = \mu_n - \mu_m$ . The eigenvalues are given by  $\pm\mu_n$ , where

$$\mu_n = \frac{1}{2(C_J + C_g)} \sqrt{\frac{8\delta^2 C_F (C_J + C_g)^2 + e^2 \omega C_J^2 (n+1)}{2C_F}}. \quad (6)$$

We denote by  $\Delta = E_J - \omega$  the detuning between the Josephson energy and cavity field frequency, ( $\delta = \Delta/2$ ). Based on the preparatory work, now we can find an exact solution under certain conditions of the whole system. With this in mind we will assume that the initial state is prepared to be a particular coherent state  $\rho^f(0) = |\alpha\rangle\langle\alpha|$  of the field with the Cooper pair box prepared in the state  $\rho^J(0) = |e\rangle\langle e|$ . The initial state of the system can be expressed in the product density matrix form,  $\rho(0) = \rho^J(0) \otimes \rho^f(0)$ . Consequently,

the general solution to equation (5) may be written explicitly as

$$\begin{aligned}
\hat{\rho}(t) = & \sum_{n,m=0}^{\infty} b_n b_m^* \exp\left(\frac{-\gamma}{2}t\right) \exp(-\gamma t(n-m)^2) \\
& \times \left\{ \exp(-i\beta_{12}) (\cos(\mu_{nm}(t)) + \cos(\mu'_{nm}(t))) |n, e\rangle \langle m, e| \right. \\
& - \frac{i}{2} \exp(-i\beta_{12}) \sin(\mu_{nm}(t)) |n, e\rangle \langle m+1, g| \\
& + \frac{i}{2} \exp(i\beta_{12}) \sin(\mu_{nm}(t)) |n+1, g\rangle \langle m, e| + \exp(-i\beta_{12}) \\
& \left. \times (\cos(\mu_{nm}(t)) - \cos(\mu'_{nm}(t))) |n+1, g\rangle \langle m+1, g| \right\}, \tag{7}
\end{aligned}$$

where  $\mu_{nm}(t) = \mu_n(t) - \mu_m(t)$ ,  $\mu'_{nm}(t) = \mu_n(t) + \mu_m(t)$ . The probability distribution among Fock states is Poissonian,  $b_n = \langle n|\alpha\rangle$ , with  $\bar{n} = |\alpha|^2$  and  $\beta_{12} = \beta - \beta^*$ , where  $\beta$  is the phase of the initial state of the field i.e.  $\alpha = |\alpha|e^{i\beta}$ . The decoherence effect on the dynamical evolution of the present system can be discussed through the phase-damping constant  $\gamma$ .

### 3 Coherence loss and entanglement

In general, due to decoherence, a pure state is apt to change into a mixed state. However, in many cases of quantum information processing, one requires a state with high purity and large amount of entanglement. Therefore, it is necessary to consider the mixture of the state and its relation with entanglement.

Here we use the idempotency defect, defined by linear entropy, as a measure of the degree of mixture for a state  $\hat{\rho}^J(t)$ , in analogy to what is done for the calculation of the entanglement in terms of von Neumann entropy [26] which has similar behavior. In order to analyze what happens to the Cooper pair box, we trace out the field variables from the state  $\hat{\rho}(t)$  and get the reduced density matrix  $\hat{\rho}^J(t) = \text{tr}_f \hat{\rho}(t)$ . The idempotency defect as a measure of coherence loss can be written as

$$\mathcal{E}_t^{(J)} = \text{Tr} \left\{ \hat{\rho}_J(t)(1 - \hat{\rho}_J(t)) \right\}, \tag{8}$$

where  $\mathcal{E}_t^{(J)}$  has a zero value for a pure state and 1 for a completely mixed state.

Supplemental to the analytical solution presented in the above section, here we discuss the results obtained numerically and interesting situations occurring for different values of the detuning and phase-damped cavity parameters. We consider the experimental parameters, described above, which are accessible using the present day technology as,  $C_J \sim 10^{-15}\text{F}$ ,  $\omega \simeq 10^{10}\text{Hz}$ ,  $C_F \sim 10^{-11}\text{F}$ ,  $K_B T \ll E_J \sim \hbar\omega \ll E_c$  and the initial state of the field as coherent state. In order to analyze the effects resulting from variation in

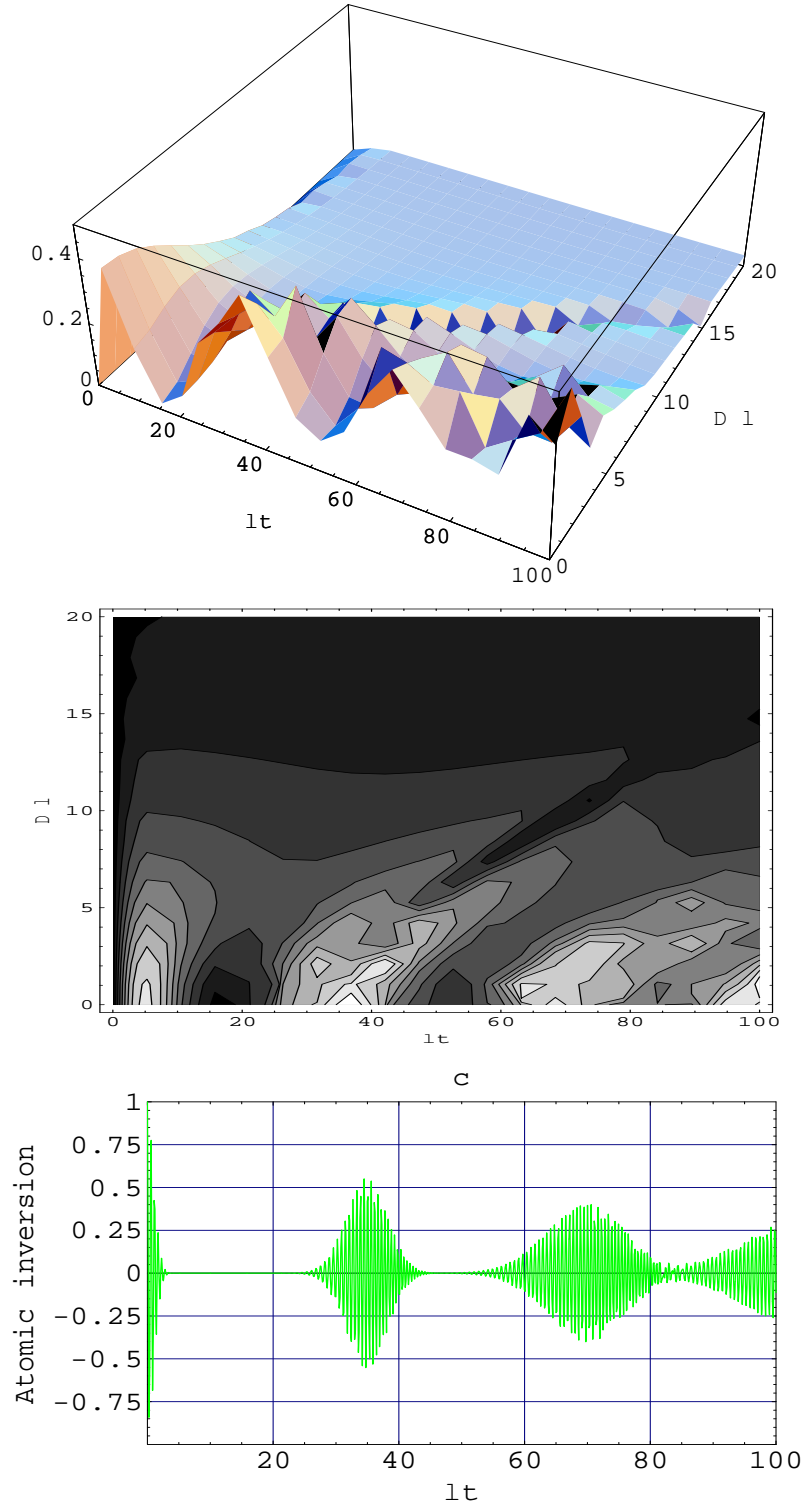


Figure 2: (a,b) Plot of idempotency defect as a function of the dimensionless scaled time  $\lambda t$  ( $\lambda = \sqrt{e^2 \omega / (\hbar C_F)}$ ) and the detuning parameter  $\Delta/\lambda$  and (c) the atomic inversion as a function of the scaled time  $\lambda t$ . In the contour plot, disentanglement is shown in the severe shading areas.

the detuning or phase damped cavity we consider the idempotency defect as a function of the scaled time  $\lambda t$  and  $\Delta/\lambda$  ( $\gamma/\lambda$ ) shown in figures 2 and 3. We have fixed the mean photon number of the coherent field as  $\bar{n} = 10$ . As can be seen from figure 1,  $\mathcal{E}_t^{(J)}$  smoothly diminishes with increasing the detuning parameter. For further increasing of the detuning the impurity of the state of the Cooper pair box system is rapidly growing and  $\mathcal{E}_t^{(J)}$  disappears completely. For the case when we take  $\gamma = \Delta = 0.0$ , we get almost zero values for the idempotency defect only at  $t = 0$ , which means that a pure state will not be reached at any time except at the initial stage of the interaction time (see figure 1). To apprehend the essential features of detuning effects on coherence loss, we presented in figure 1b the contour plot of the concurrence, where complete separable states are shown in the severe shading areas.

In Figure 2c we show the time evolution of the atomic inversion. Apparently, it is easy to observe the existence of collapse and revival of Rabi oscillations of the atomic inversion and the first maximum of the idempotency defect is achieved in the collapse time, while at one-half of the revival time, the idempotency defect reaches its local minimum. Also, it is noticed that in absence of both detuning and phase-damping, a gradual decrease in the amplitudes of the Rabi oscillations is shown.

On the other hand, the decoherence introduces irreversibility into the Junction dynamics and also on the global system. During the repeated periods of maximum and minimum idempotency defect, the states of the Junction and field lose and gain coherence, but given the continuous amplitude decreasing of coherent states, the coherence recovered by the Junction is never that which was lost. We may refer here to the work given in reference [27] where engineering maximally entangled states has been discussed for different systems. Of course, larger the value of  $\gamma$ , the more rapid is this phenomenon in the sense of the idempotency defects being close to one (the purity loss of the junction state is complete). In particular, for the limiting case of large  $\gamma$  ( $\gamma = 0.1\lambda$ ), the idempotency defect blows up from zero and rapidly saturates i.e. as time goes on a long-lived coherence loss is observed (see figure 2 (top)). Even in a weak-damping cavity, the difference between consecutive local maximum and minimum diminishes with time, since idempotency defect tends to asymptotic values. Speaking specifically, it arrives at a maximum value (about 1) at large values of the phase damping parameter, and then remains nearly invariant regardless of the increase of time or  $\gamma$ , while the idempotency defect always remains vanishing at  $\lambda t = 0$ .

We can gain further physical insight into the dynamical effect of the phase damping by considering the general case (mixed state entanglement). To measure the degree of entanglement for mixed states of bipartite systems composed by two-level subsystems,



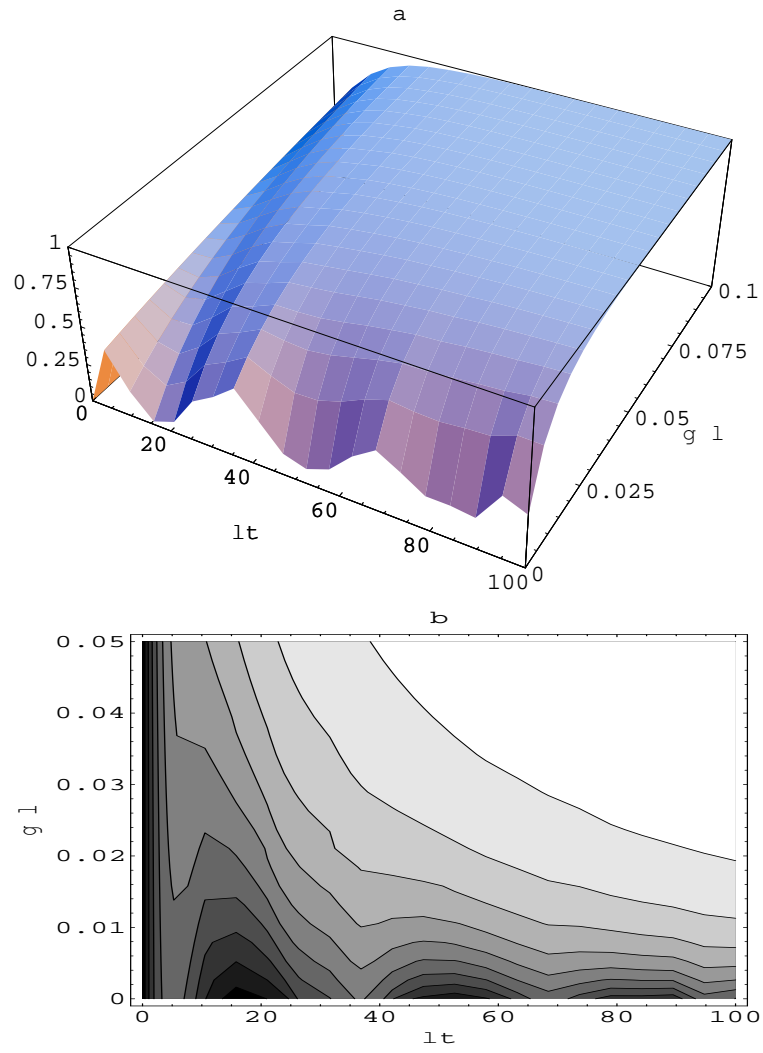


Figure 3: Plot of idempotency defect as a function of the scaled time  $\lambda t$  and the decoherence parameter  $\gamma/\lambda$ . In the contour plot, a complete mixture is shown in the non-shaded area.

one needs to consider a commonly used measure such as the concurrence [28] which has been proven to be a reasonable entanglement measure or negativity [29]. Analysis of the entanglement decay rates under decoherence for different models of the interaction between systems of arbitrary dimensions with the environment has been presented [8]. For the density matrix  $\hat{\rho}(t)$ , which represents the state of a bipartite system, concurrence is defined as

$$C(\hat{\rho}) = \max\{0, \Re_1 - \Re_2 - \Re_3 - \Re_4\}, \quad (9)$$

where the  $\Re_i$  are the non-negative eigenvalues, in decreasing order ( $\Re_1 \geq \Re_2 \geq \Re_3 \geq \Re_4$ ), of the Hermitian matrix  $\hat{\Upsilon} \equiv \sqrt{\sqrt{\tilde{\rho}}\tilde{\rho}\sqrt{\tilde{\rho}}}$  and  $\tilde{\rho} = (\hat{\sigma}_y \otimes \hat{\sigma}_y) \hat{\rho}^* (\hat{\sigma}_y \otimes \hat{\sigma}_y)$ . Here,  $\hat{\rho}^*$  represents the complex conjugate of the density matrix  $\hat{\rho}$  when it is expressed in a fixed basis and  $\hat{\sigma}_y$  represents the Pauli matrix in the same basis. The function  $C(\hat{\rho})$  ranges from 0 for a separable state to 1 for a maximum entanglement.

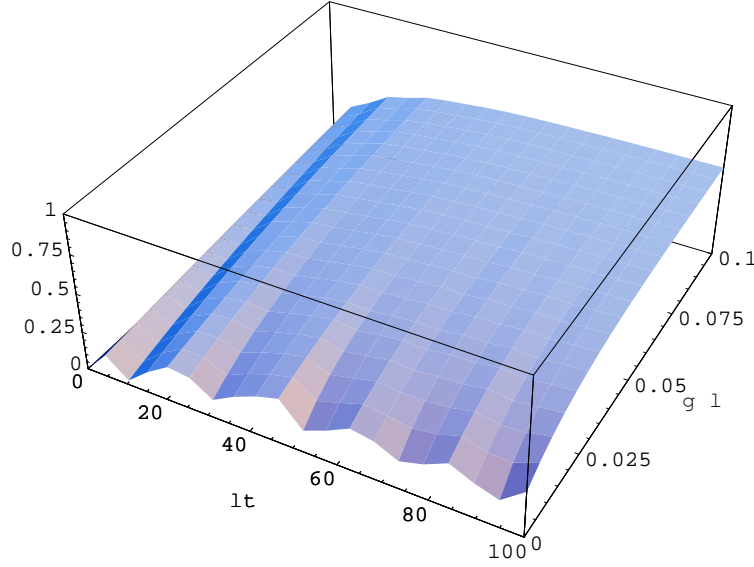


Figure 4: Development of the concurrence  $C(\hat{\rho})$  as a function of the scaled time  $\lambda t$  and the decoherence parameter  $\gamma/\lambda$ . The other parameters are the same as figure 3.

In figure (4), we plot the numerically evaluated results for the concurrence  $C(\hat{\rho})$ , as a function of the scaled time  $\lambda t$  and phase damping parameter  $\gamma$  in units of  $\lambda$ . As confirmed in figure (4), the asymptotic value of the concurrence is obtained when the phase damping is increased. Of course, there are some differences between the concurrence and idempotency defect in the amplitudes but the general behavior is the same i.e. comparing figures 3a and 4, one can find that concurrence results in qualitative analogy with the results of the idempotency defect. This may be thought to arise from the asymptotic limits

which have been observed in both figures 3 and 4 due to the phase damping. Although the entanglement, as witnessed by the concurrence, is lower than maximal possible (about 1), it has a fixed value as the phase damping increased (long-lived entanglement). We have confirmed the predictions of this phenomenon using a systematic numerical analysis where a number of relevant parameters has been varied. However, once the initial state setting of the Cooper pair box is considered as  $\rho^J(0) = \cos^2(\theta)|e\rangle\langle e| + \sin^2(\theta)|g\rangle\langle g|$ , ( $0 < \theta < \pi/2$ ), this feature no longer exists and entanglement vanishes in an asymptotic limit.

Obviously, the above novel phenomena are directly related to the recent results of Ref. [18]. With this at hand, one may envision quantum computers using these long-lived entangled states for quantum memory and for extended quantum information processing [30] where, superconducting single Cooper pair boxes using superconducting single electron transistor fabricated on the same chip as an electrometer has been presented in Ref. [31] and the electronic control of a single-qubit achieved in a solid state device has been demonstrated [32]. In these works, it has been shown that, the general scalability of such a solid state device will be a prerequisite for a practical quantum computer. Also, it has been shown that [30] only twice the resources (qubits + elementary quantum gates in the decoherence free subspace) are needed to realize up to 4 orders of magnitude more operations before the quantum information is lost to the environment.

It has been predicted only recently that the one-body and two-body responses to a noisy environment can follow surprisingly different pathways to complete decoherence [33, 34]. The first experimental work and impressive results in this new domain have been reported in Ref. [35]. They have devised an elegantly clean way to check and to confirm the existence of so-called entanglement sudden death, a two-body disentanglement that is novel among known relaxation effects because it has no lifetime in any usual sense, that is, entanglement terminates completely after a finite interval, without a smoothly diminishing long-time tail [36, 37].

## 4 Conclusion

In conclusion, we suggest that applying a microwave field to a Cooper-pair box via the gate capacitance, a long-lived entanglement can be realized i.e. the Cooper pair composite system is entangled forever. In our work we have extended the exactly solvable model of a single-Cooper-pair box model by taking into account the decoherence effect on the purity loss and entanglement. Decoherence is a very useful concept that has recently been widely investigated and has turned out to be very prolific. It is intuitively related to the loss of purity of a final state of the quantum system. However, it is demonstrated

that, it is not correct to think that a quantum system, by increasing the decoherence, will suffer an increasing loss of quantum coherence. It is worth stressing in this respect, an appropriate choice of the system parameters, specifically, large values of the decoherence parameter and initial state setting of the Cooper pair box does give an interesting effect to the entanglement process as a long-lived entanglement which may lead to unexpected applications.

We are sure that our ground breaking work on the dynamics of quantum entanglement in the Cooper pair box system exactly, will lead both to understand the generic behaviors of these systems by model studies and to add more features to the theoretical models that can provide a closer depiction of reality, captured in the near future by higher precision experiments. A topic that remains open in almost all decoherence discussions, however, is the preservation or destruction of two-body quantum coherence when both bodies are small. We are convinced that future experiments exploiting the particular advantages of these models will reveal interesting new phenomena and show many surprises.

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